Quiz 4 - Sections 6 and 9

Fall 2012

(We will omit the arrows above vectors when there is no confusion with scalars.)

1. (4 points) Set up the integral for the arc length of the curve $r(t) = \cos^3 t\,\vec{i} - \sin^3 t\,\vec{j}$

Solution:

We first compute the derivative vector (using the product rule):

$$r't(t) = -3\cos^2 t \sin t\,\vec{i} - 3\sin^2 t \cos t\,\vec{j},$$

next we find its length:

$$\begin{aligned} |r'(t)| &= 3\sqrt{\cos^4 t \sin^2 t + \sin^4 t \cos^2 t} \\ &= 3\sqrt{\cos^2 t \sin^2 t(\cos^2 t + \sin^2 t)} \\ &= 3|\cos t||\sin t|. \end{aligned}$$

Therefore the integral for the arc length is

$$\int_{t_0}^{t_1} 3|\cos t||\sin t|\,dt$$

2. (6 points) For the function $f(x,y) = \frac{x^2-y^2}{x-y}$ do the following:

(a) Find the domain of the function.

Solution: The only dangerous operation here is division, so we only have to find the points where the denominator isn't 0. But this is easy since this is just the set $\{(x,y) : x \neq y\}$.

(b) Compute the level curve of the function through the point $(1,0)$.

Solution: We have to find all points (x,y) such that $f(x,y) = f(1,0) = 1$. To do this we just try to simplify the equation

$$\frac{x^2-y^2}{x-y} = 1.$$

First notice that we can factor the numerator as $(x-y)(x+y)$, and since we are always working in the domain of f, $x-y$ is never 0 so we can factor this and get the equivalent equation

$$x+y=1,$$

so the level curve is the line through $(1,0)$ of slope -1.

(b) Compute the limit of the function as $(x,y) \to (1,1)$.

Solution: We are outside the domain of f so we cannot just plug $(1,1)$ into f. Since $x \neq y$ we can factor f as in part (b) and now the limit is reduced to:

$$\lim_{(x,y)\to(1,1)} f(x,y) = \lim_{(x,y)\to(1,1)} x+y. \tag{1}$$

But now the function $x+y$ is continuous so we can just plug $(1,1)$ in $x+y$ and obtain that the limit is 2.