Quiz 3 - Section 9

Fall 2012

(We will omit the arrows above vectors when there is no confusion with scalars.)

1. (6 points) The acceleration vector of a particle in space at time t is given by $a(t) = e^{-t}\overrightarrow{i} + (2t^3+1)\overrightarrow{j} - \sin t\,\overrightarrow{k}$, and at time $t = 0$ we have $v(0) = \overrightarrow{i} - 2\overrightarrow{k}$, $r(0) = \overrightarrow{i} + \overrightarrow{j}$. Find the position vector $r(t)$.

Solution:

From the Fundamental Theorem of Calculus we have:

$$
\begin{aligned}
v(t) &= v(0) + \int_0^t a(s), ds && (1)\\
&= \langle 1, 0, -2\rangle + \int_0^t \langle e^{-s}, 2s^3+1, -\sin s\rangle\, ds && (2)\\
&= \langle 2 - e^{-t}, \frac{1}{2}t^4 + t, \cos t - 3\rangle. && (3)
\end{aligned}
$$

Now we can find r using the same technique:

$$
\begin{aligned}
r(t) &= \langle 1, 1, 0\rangle + \langle \int_0^t 2 - e^{-s}\, ds, \int_0^t \frac{1}{2}s^4 + s\, ds, \int_0^t \cos s - 3\, ds\rangle && (4)\\
&= \left(2t - e^{-t}\right)\overrightarrow{i} + \left(1 + \frac{1}{10}t^5 + \frac{1}{2}t^2\right)\overrightarrow{j} + \left(\sin t - 3t\right)\overrightarrow{k} && (5)
\end{aligned}
$$

$$\boxed{\left(2t - e^{-t}\right)\overrightarrow{i} + \left(1 + \tfrac{1}{10}t^5 + \tfrac{1}{2}t^2\right)\overrightarrow{j} + \left(\sin t - 3t\right)\overrightarrow{k}}$$

2. (2+2=4 points)

   a) Write down the "dot product rule" for the derivative of $\overrightarrow{u(t)} \cdot \overrightarrow{v(t)}$

   b) Using the rule in part a) compute $\frac{d}{dt}\left(|v(t)|^2\right)$

Solution: The "dot product rule" for the derivative of the dot product of two vectors is

$$u'(t) \cdot v(t) + u(t) \cdot v'(t).$$

This is part a) of the solution.

To find the derivative of $|v(t)|^2$ we recall that the square of the length of a vector is just the dot product of the vector with itself, therefore

$$\frac{d}{dt}|v(t)|^2 = \frac{d}{dt}\big(v(t) \cdot v(t)\big) = 2v'(t) \cdot v(t)$$