

# Quiz 1 - Section 6

Fall 2012

1. (6 points) Write  $\vec{u}$  as a sum of a vector parallel to  $\vec{v}$  and another vector perpendicular to  $\vec{v}$ , where  $u = \vec{j} + k$  and  $\vec{v} = -\vec{i} + \vec{k}$ . Namely write  $\vec{u} = \vec{u}_{\parallel} + \vec{u}_{\perp}$  for appropriate vectors  $\vec{u}_{\parallel}$  and  $\vec{u}_{\perp}$ .

**Solution:** (We will omit the arrows above vectors when there is no confusion with scalars.)

The parallel vector,  $u_{\parallel}$  must be  $\text{proj}_v(u)$ , so

$$u_{\parallel} = \frac{u \cdot v}{|v|^2} v = \frac{1}{2} v = -\frac{1}{2} \vec{i} + \frac{1}{2} \vec{k}.$$

Now, if  $u = u_{\parallel} + u_{\perp}$ , then  $u_{\perp}$  should be  $u - u_{\parallel}$ , indeed, if we define  $u_{\perp}$  in this way then  $u_{\perp} = \frac{1}{2} \vec{i} - \vec{j} + \frac{1}{2} \vec{k}$  and we easily check that  $u_{\perp} \cdot v = 0$ , that is, they are perpendicular as they should.

$$\begin{aligned} u_{\parallel} &= -\frac{1}{2} \vec{i} + \frac{1}{2} \vec{k} \\ u_{\perp} &= \frac{1}{2} \vec{i} - \vec{j} + \frac{1}{2} \vec{k} \end{aligned}$$

2. (4 points) Compute the area of the parallelogram determined by the points  $P(-1, 0, 0)$ ,  $Q(1, 0, -1)$ ,  $R(0, 1, -2)$ , with sides  $\vec{PQ}$  and  $\vec{PR}$ .

**Solution:** First we compute  $\vec{PQ}$  and  $\vec{PR}$ :

$$\begin{aligned} \vec{PQ} &= \langle 2, 0, -1 \rangle \\ \vec{PR} &= \langle 1, 1, -2 \rangle. \end{aligned}$$

Now we just have to take their cross product and then its length:

$$\vec{PQ} \times \vec{PR} = \begin{vmatrix} i & j & k \\ 2 & 0 & -1 \\ 1 & 1 & -2 \end{vmatrix} = \langle 1, 3, 2 \rangle,$$

which has length  $\sqrt{14}$ .

Answer:  $\sqrt{14}$  units

3. (3 points) A projectile is fired from the ground at a speed of 1440 ft/sec in the direction of the vector  $\langle -\sqrt{7}, \sqrt{7} \rangle$ . Express the velocity vector as a product of the *speed* and a *unit vector*. Please use the vectors  $\vec{i}$ ,  $\vec{j}$  and  $\vec{k}$  in your answer.

**Solution:** The speed was given in the statement: 1440 feet/sec, so we just have to compute the unit direction vector.

The direction vector is  $\vec{v} = \langle -\sqrt{7}, \sqrt{7} \rangle$ , so its length is

$$\sqrt{7+7} = \sqrt{14}.$$

To obtain the unit direction vector we just divide by its length:

$$\begin{aligned}\vec{u} &= \frac{1}{\sqrt{14}} \vec{v} = \left\langle \frac{-\sqrt{7}}{\sqrt{14}}, \frac{\sqrt{7}}{\sqrt{14}} \right\rangle \\ &= \left\langle \frac{-1}{\sqrt{2}}, \frac{1}{\sqrt{2}} \right\rangle.\end{aligned}$$

So the solution would be

$$\vec{v} = 1440 \cdot \left( \frac{-1}{\sqrt{2}} \vec{i} + \frac{1}{\sqrt{2}} \vec{j} \right)$$

Speed = 1440ft/sec Unit vector = $\frac{-1}{\sqrt{2}} \vec{i} + \frac{1}{\sqrt{2}} \vec{j}$
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