Quiz 1 - Section 6

Fall 2012

1. (6 points) Write $\overrightarrow{u}$ as a sum of a vector parallel to $\overrightarrow{v}$ and another vector perpendicular to $\overrightarrow{v}$, where $u = -\overrightarrow{j} + k$ and $\overrightarrow{v} = \overrightarrow{i} + \overrightarrow{k}$. Namely write $\overrightarrow{u} = \overrightarrow{u}_{\parallel} + \overrightarrow{u}_{\perp}$ for appropriate vectors $\overrightarrow{u}_{\parallel}$ and $\overrightarrow{u}_{\perp}$.

Solution: (We will omit the arrows above vectors when there is no confusion with scalars.)

The parallel vector, $u_{\parallel}$ must be $\text{proj}_v(u)$, so

$$u_{\parallel} = \frac{u \cdot v}{|v|^2} v = \frac{1}{2} v = \frac{1}{2}\overrightarrow{i} + \frac{1}{2}\overrightarrow{k}.$$

Now, if $u = u_{\parallel} + u_{\perp}$, then $u_{\perp}$ should be $u - u_{\parallel}$, indeed, if we define $u_{\perp}$ in this way then $u_{\perp} = -\frac{1}{2}\overrightarrow{i} - \overrightarrow{j} + \frac{1}{2}\overrightarrow{k}$ and we easily check that $u_{\perp} \cdot v = 0$, that is, they are perpendicular as they should.

$$\boxed{\begin{array}{l} u_{\parallel} = \frac{1}{2}\overrightarrow{i} + \frac{1}{2}\overrightarrow{k} \\ u_{\perp} = -\frac{1}{2}\overrightarrow{i} - \overrightarrow{j} + \frac{1}{2}\overrightarrow{k} \end{array}}$$

2. (4 points) Compute the area of the parallelogram determined by the points $P(-1, 1, 0)$, $Q(1, 1, -1)$, $R(0, 2, -2)$, with sides $\overrightarrow{PQ}$ and $\overrightarrow{PR}$.

Solution: First we compute $\overrightarrow{PQ}$ and $\overrightarrow{PR}$:

$$\overrightarrow{PQ} = \langle 2, 0, -1 \rangle$$
$$\overrightarrow{PR} = \langle 1, 1, -2 \rangle.$$

Now we just have to take their cross product and then its length:

$$\overrightarrow{PQ} \times \overrightarrow{PR} = \begin{vmatrix} i & j & k \\ 2 & 0 & -1 \\ 1 & 1 & -2 \end{vmatrix} = \langle 1, 3, 2 \rangle,$$

which has length $\sqrt{14}$.

$$\boxed{\text{Answer: } \sqrt{14} \text{ units}}$$

3. (3 points) A projectile is fired from the ground at a speed of 1440 ft/sec in the direction of the vector $\langle -\sqrt{7}, \sqrt{7} \rangle$. Express the velocity vector as a product of the *speed* and a *unit vector*. Please use the vectors $\vec{i}$, $\vec{j}$ and $\vec{k}$ in your answer.

Solution: The speed was given in the statement: 1440 feet/sec, so we just have to compute the unit direction vector.

The direction vector is $\vec{v} = \langle -\sqrt{7}, \sqrt{7} \rangle$, so its length is

$$\sqrt{7+7} = \sqrt{14}.$$

To obtain the unit direction vector we just divide by its length:

$$\begin{aligned} \vec{u} &= \frac{1}{\sqrt{14}} \vec{v} = \left\langle \frac{-\sqrt{7}}{\sqrt{14}}, \frac{\sqrt{7}}{\sqrt{14}} \right\rangle \\ &= \left\langle \frac{-1}{\sqrt{2}}, \frac{1}{\sqrt{2}} \right\rangle. \end{aligned}$$

So the solution would be

$$\vec{v} = 1440 \cdot \left(\frac{-1}{\sqrt{2}} \vec{i} + \frac{1}{\sqrt{2}} \vec{j} \right)$$

Speed = 1440ft/sec
Unit vector = $\frac{-1}{\sqrt{2}} \vec{i} + \frac{1}{\sqrt{2}} \vec{j}$