Quiz 1 - Section 9

Fall 2012

1. (4 points) Compute the center and radius of the sphere with equation $x^2 + y^2 + z^2 + 2x - 6y + 5z - 1 = 0$

Solution: First we complete squares:

$$x^2 + 2x + 1 - 1 \quad + \quad y^2 - 6y + 9 - 9 \quad + \quad z^2 + 5z + \left(\frac{5}{2}\right)^2 - \left(\frac{5}{2}\right)^2 = 0$$

$$(x+1)^2 + (y-3)^2 + \left(z + \frac{5}{2}\right)^2 = 1 + 1 + 9 + \left(\frac{5}{2}\right)^2$$

$$(x+1)^2 + (y-3)^2 + \left(z + \frac{5}{2}\right)^2 = \frac{69}{4}$$

Now the center is $C\left(-1, 3, -\frac{5}{2}\right)$ and the radius is $\sqrt{\frac{69}{4}} = \frac{\sqrt{69}}{2}$.

$$\boxed{\begin{array}{l}\text{Center} = \left(-1, 3, -\frac{5}{2}\right) \\ \text{Radius} = \frac{\sqrt{69}}{2}\end{array}}$$

2. (3 points) Given $A(1, -3, 2)$ and $B(-5, -1, 2)$, $C(-3, 0, 4)$ and $D(1, -1, 0)$, compute the vector $-\overrightarrow{AB} + 3\overrightarrow{CD}$. Please use the vectors $\overrightarrow{i}$, $\overrightarrow{j}$ and $\overrightarrow{k}$ in your answer.

Solution: First we compute $\overrightarrow{AB}$ and $\overrightarrow{CD}$:

$$\overrightarrow{AB} = \overrightarrow{B} - \overrightarrow{A} = \langle -5 - 1, -1 - -3, 2 - 2\rangle = \langle -6, 2, 0\rangle$$
$$\overrightarrow{CD} = \overrightarrow{D} - \overrightarrow{C} = \langle 1 - -3, -1 - 0, 0 - 4\rangle = \langle 4, -1, -4\rangle.$$

Now it's easy:

$$\begin{aligned} -\overrightarrow{AB} + 3\overrightarrow{CD} &= -\langle -6, 2, 0\rangle + 3\langle 4, -1, -4\rangle \\ &= \langle 6 + 12, -2 - 3, 0 - 12\rangle \\ &= \langle 18, -5, -12\rangle, \end{aligned}$$

so the solution would be: $18\overrightarrow{i} - 5\overrightarrow{j} - 12\overrightarrow{k}$.

$$\boxed{\text{Answer: } 18\overrightarrow{i} - 5\overrightarrow{j} - 12\overrightarrow{k}}$$

3. (3 points) A projectile is fired from the ground at a speed of 2718 ft/sec in the direction of the vector $\langle\sqrt{2}, -\sqrt{8}\rangle$. Express the velocity vector as a product of the *speed* and a *unit vector*. Please use the vectors $\vec{i}$, $\vec{j}$ and $\vec{k}$ in your answer.

Solution: The speed was given in the statement: 2719 feet/sec, so we just have to compute the unit direction vector.

The direction vector is $\vec{v} = \langle\sqrt{2}, -\sqrt{8}\rangle$, so its length is

$$\sqrt{2+8} = \sqrt{10}.$$

To obtain the unit direction vector we just divide by its length:

$$\begin{aligned}\vec{u} &= \frac{1}{\sqrt{10}}\vec{v} = \Big\langle \frac{\sqrt{2}}{\sqrt{10}}, \frac{\sqrt{8}}{\sqrt{10}} \Big\rangle \\ &= \Big\langle \frac{1}{\sqrt{5}}, -\frac{2}{\sqrt{5}} \Big\rangle.\end{aligned}$$

So the solution would be

$$\vec{v} = 2718 \cdot \Big(\frac{1}{\sqrt{5}}\vec{i} - \frac{2}{\sqrt{5}}\vec{j} \Big)$$

Speed = 2718ft/sec
Unit vector = $\frac{1}{\sqrt{5}}\vec{i} - \frac{2}{\sqrt{5}}\vec{j}$