

Quiz 1 - Section 6

Fall 2012

1. (4 points) Compute the center and radius of the sphere with equation $x^2 + y^2 + z^2 + 8x - 6y + 4z - 7 = 0$

Solution: First we complete squares:

$$\begin{aligned}x^2 + 8x + 16 - 16 + y^2 - 6y + 9 - 9 + z^2 + 4z + 4 - 4 &= 7 \\(x + 4)^2 + (y - 3)^2 + (z + 2)^2 &= 7 + 16 + 9 + 4 \\(x + 4)^2 + (y - 3)^2 + (z + 2)^2 &= 36.\end{aligned}$$

Now the center is $C(-4, 3, -2)$ and the radius is $\sqrt{36} = 6$.

Center = $(-4, 3, -2)$
Radius = 6

2. (3 points) Given $A(-7, 5, 2)$ and $B(-5, 1, 0)$, $C(4, 1, 8)$ and $D(2, -1, 1)$, compute the vector $-3\overrightarrow{AB} + 5\overrightarrow{CD}$. Please use the vectors \vec{i} , \vec{j} and \vec{k} in your answer.

Solution: First we compute \overrightarrow{AB} and \overrightarrow{CD} :

$$\begin{aligned}\overrightarrow{AB} &= \vec{B} - \vec{A} = \langle -5 + 7, 1 - 5, 0 - 2 \rangle = \langle 2, -4, -2 \rangle \\ \overrightarrow{CD} &= \vec{D} - \vec{C} = \langle 2 - 4, -1 - 1, 1 - 8 \rangle = \langle -2, -2, -7 \rangle.\end{aligned}$$

Now it's easy:

$$\begin{aligned}-3\overrightarrow{AB} + 5\overrightarrow{CD} &= -3\langle 2, -4, -2 \rangle + 5\langle -2, -2, -7 \rangle \\ &= \langle -6 - 10, 12 - 10, 6 - 35 \rangle \\ &= \langle -16, 2, -29 \rangle,\end{aligned}$$

so the solution would be: $-16\vec{i} + 2\vec{j} - 29\vec{k}$.

Answer: $-16\vec{i} + 2\vec{j} - 29\vec{k}$

3. (3 points) A projectile is fired from the ground at a speed of 1440 ft/sec in the direction of the vector $\langle -\sqrt{7}, \sqrt{7} \rangle$. Express the velocity vector as a product of the *speed* and a *unit vector*. Please use the vectors \vec{i} , \vec{j} and \vec{k} in your answer.

Solution: The speed was given in the statement: 1440 feet/sec, so we just have to compute the unit direction vector.

The direction vector is $\vec{v} = \langle -\sqrt{7}, \sqrt{7} \rangle$, so its length is

$$\sqrt{7+7} = \sqrt{14}.$$

To obtain the unit direction vector we just divide by its length:

$$\begin{aligned}\vec{u} &= \frac{1}{\sqrt{14}} \vec{v} = \left\langle \frac{-\sqrt{7}}{\sqrt{14}}, \frac{\sqrt{7}}{\sqrt{14}} \right\rangle \\ &= \left\langle \frac{-1}{\sqrt{2}}, \frac{1}{\sqrt{2}} \right\rangle.\end{aligned}$$

So the solution would be

$$\vec{v} = 1440 \cdot \left(\frac{-1}{\sqrt{2}} \vec{i} + \frac{1}{\sqrt{2}} \vec{j} \right)$$

Speed = 1440ft/sec Unit vector = $\frac{-1}{\sqrt{2}} \vec{i} + \frac{1}{\sqrt{2}} \vec{j}$
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