Calculus III - Quiz 3 - Spring 2015

Name: ____________________________________

1. (3 points) Let $\mathbf{r}(t)$ be some vector function and suppose that $\mathbf{r}'(t_0) \neq 0$, then $\mathbf{r}'(t_0)$ is:

   1. Perpendicular to the curve at $\mathbf{r}(t_0)$.
   2. Tagent to the curve at $\mathbf{r}(t_0)$.
   3. The length of the curve up to time t_0.

   Solution: The answer is (2).

2. (4 points) Let f be the function of two variables defined by

$$f(x,y) = \begin{cases} \frac{x^3y - y^3x}{xy} & \text{if } x \text{ and } y \text{ are not } 0 \\ 1 & \text{if } x \text{ or } y \text{ are } 0. \end{cases}$$

   The function f is:

   1. Continuous at $(0,0)$.
   2. Not continuous at $(0,0)$ but continuous everywhere else.
   3. None of the above.

   Solution: The answer is (3) since the function is not continuous whenever $y = 0$ or $x = 0$. To see that f is not continuous, say at $(3,0)$, one can approximate this point using two lines: along the x axis one would get that the limit is 1 because f is always 1 there by definitions.

   One can also compute

$$\lim_{y\to 0} f(3,y) = \lim_{y\to 0} \frac{27y - 3y^3}{y} = \lim_{y\to 0} 27 - 3y^2 = 27 \neq 0.$$

   Since the limit along two different curves is different, the limit cannot exist and hence the function is discontinuous there.

3. (3 points) The level curves of $f(x, y) = \log(1 + \sqrt{x^2 + y^2})$ look like:

1. Spheres.
2. Paraboloids.
3. Circles.

Solution: One can answer this question without actually figuring out what the level curves of f are, just by elimination: curves cannot be 2-dimensional so (1) and (2) cannot be true, so it must be (3).

One can also just compute the level curves: for $C \geq 0$

$$\begin{aligned} \log(1 + \sqrt{x^2 + y^2}) &= C \\ 1 + \sqrt{x^2 + y^2} &= e^C \\ x^2 + y^2 &= (e^C - 1)^2. \end{aligned}$$

And these are circles centered at the origin of radii $e^C - 1$. If $C < 0$ then there are no level curves since $\log(1 + \sqrt{x^2 + y^2}) \geq 0$.