Calculus III - Spring 2015

January 22, 2015

Name: ______________________________

1. (3 points) Find the center and radius of the the sphere with the following equation:

$$x^2 + y^2 + z^2 - 2x - 4y - 6z - 2 = 0.$$

Solution: We can complete the square for each part separately:

$$x^2 - 2x = (x-1)^2 - 1 \quad (1)$$
$$y^2 - 4y = (y-2)^2 - 4 \quad (2)$$
$$z^2 - 6z = (z-3)^2 - 9. \quad (3)$$

So, after rearranging the terms, we obtain:

$$(x-1)^2 + (y-2)^2 + (z-3)^2 = 16 = 4^2,$$

so the center is the point $(1, 2, 3)$ and the radius is 4.

2. (3 points) Find the projection vector of $\vec{u}$ onto $\vec{v}$, where

$$\vec{u} = \langle 3, 4 \rangle$$
$$\vec{v} = 5\vec{i} - \vec{j}$$

Solution: The formula for the projection is

$$\text{proj}_v(u) = \frac{u \cdot v}{|v|^2} v,$$

so we need to compute each term.
For the dot product:

$$u \cdot v = 3 \cdot 5 + 4 \cdot (-1) = 11.$$

The length of v squared is just $5^2 + 1^1 = 26$, so the resulting vector is

$$\left\langle \frac{55}{26}, \frac{-11}{26} \right\rangle.$$

3. (4 points) Find the area of the parallelogram with vertices:

$$A(2,2), B(5,1), C(6,3) \text{ and } D(3,4).$$

Solution: The two generating sides are AB and AD, so let's compute each of these vectors:

$$\vec{AB} = \langle 3, -1 \rangle$$

and

$$\vec{AD} = \langle 1, 2 \rangle.$$

The length of the cross product of two vectors is the area of the parallelogram that they generate, so (after making these vectors 3-dimensional) we get:

$$\text{Area} = |u \times v|.$$

To find the cross product we follow the usual steps:

$$u \times v = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ 3 & -1 & 0 \\ 1 & 2 & 0 \end{vmatrix} = 0 \cdot \vec{i} + 0 \cdot \vec{j} + 7 \cdot \vec{k}.$$

The length of this vector is 7, so the area must be 7.