Math 2374 - Quiz 7

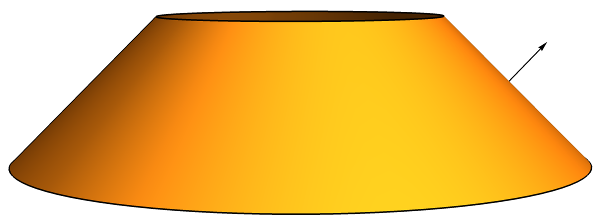
Name: ______________________________

Section: ______

Let S be the following piece of an inverted cone

$$z = 1 - \sqrt{x^2 + y^2} \qquad\qquad 0 \leq z \leq \frac{1}{2}.$$

We give S the orientation for which the normal vector $\vec{n}$ points *out* of the cone.



1. This surface has two disconnected boundaries: the top and bottom circles. What is the orientation on these two circles induced by the orientation of S?

Solution: The top circle is oriented clockwise when seen from above, and the bottom circle is oriented counterclockwise when seen from above.

Consider the following parametrization of S:

$$\Phi(z,\theta) = \big((1-z)\cos\theta, (1-z)\sin\theta, z\big),$$

for $z \in [0, 1/2]$ and $\theta \in [0, 2\pi]$.

2. Compute $T_z \times T_\theta$. Is Φ orientation preserving or reversing? Explain your answer.

Solution:

$$T_z = (-\cos\theta, -\sin\theta, 1) \quad \text{and} \quad T_\theta = (-(1-z)\sin\theta, (1-z)\cos\theta, 0),$$

so

$$T_z \times T_\theta = -(1-z)\big(\cos\theta, \sin\theta, 1\big).$$

This is pointing *inside* the cone, so the parametrization is orientation reversing.

3. Integrate the vector field $V = x\vec{i} + y\vec{j}$ over S

Solution: We have already computed $T_z \times T_\theta$, and we know Φ is orientation reversing, so

$$\begin{aligned}
\iint_S (x,y,0)\cdot dS &= -\iint_D ((1-z)\cos\theta, (1-z)\sin\theta, 0)\cdot T_z \times T_\theta \, dz\, d\theta \\
&= -\iint_D ((1-z)\cos\theta, (1-z)\sin\theta, 0)\cdot(-(1-z)(\cos\theta, \sin\theta, 1))\, dz\, d\theta \\
&= \iint_D (1-z)^2(\cos^2\theta + \sin^2\theta)\, dz\, d\theta \\
&= \iint_D (1-z)^2\, dz\, d\theta \\
&= \int_0^{2\pi}\int_0^{1/2} (1-z)^2\, dz\, d\theta \\
&= 2\pi \int_0^{1/2} (1-z)^2\, dz \\
&= \frac{7\pi}{12}.
\end{aligned}$$