Math 2374 - Quiz 3

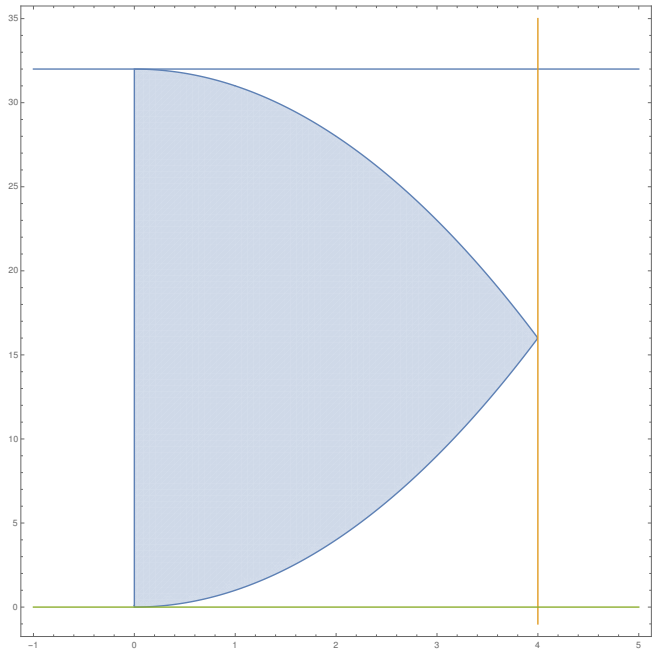
Name: _____

Section: _____

1. Sketch the domain D bounded by $x \geq 0$, $y \geq x^2$, and $y \leq 32 - x^2$ and compute

$$\iint_D x \, dx \, dy$$

Solution: The domain looks like this:



To integrate the function we use vertical slices (since it's the most natural for this domain). The range of the x variable starts at 0 and ends at (x-coordinate) of the intersection of the curves

$$y = x^2 \quad \text{and} \quad y = 32 - x^2,$$

which intersect at $x = 4$. So we can compute the integral by reducing to an interated integral as follows:

$$\begin{aligned}
\iint_D x\,dx\,dy &= \int_0^4 \int_{x^2}^{32-x^2} x\,dy\,dx \\
&= \int_0^4 x \int_{x^2}^{32-x^2} 1\,dy\,dx \\
&= \int_0^4 x(32 - x^2 - x^2)\,dx \\
&= \int_0^4 32x - 2x^3\,dx \\
&= \left[16x^2 - \frac{1}{2}x^4\right]_{x=0}^{x=4} \\
&= 2^7.
\end{aligned}$$