

Math 2374 - Quiz 2

Name: _____

Section: _____

Consider the function of three variables $f : \mathbb{R}^3 \rightarrow \mathbb{R}$ given by

$$f(x, y, z) = \det \begin{pmatrix} 1 & x & x^2 \\ 1 & y & y^2 \\ 1 & z & z^2 \end{pmatrix}.$$

1. (50 points) Write an explicit formula for f (expand the determinant).

Solution: One can compute the determinant going down the first column:

$$\begin{aligned} \det \begin{pmatrix} 1 & x & x^2 \\ 1 & y & y^2 \\ 1 & z & z^2 \end{pmatrix} &= \det \begin{pmatrix} y & y^2 \\ z & z^2 \end{pmatrix} - \det \begin{pmatrix} x & x^2 \\ z & z^2 \end{pmatrix} + \det \begin{pmatrix} x & x^2 \\ y & y^2 \end{pmatrix} \\ &= yz^2 - y^2z - xz^2 + x^2z + xy^2 - x^2y. \end{aligned}$$

2. (50 points) Compute $\frac{\partial f}{\partial x}$.

Solution: To compute $\frac{\partial f}{\partial x}$ treat everything except x as a constant:

$$\begin{aligned}\frac{\partial f}{\partial x} &= \frac{\partial}{\partial x}(yz^2 - y^2z - xz^2 + x^2z + xy^2 - x^2y) \\ &= \frac{\partial}{\partial x}(-xz^2 + x^2z + xy^2 - x^2y) \\ &= -z^2 + 2xz + y^2 - 2xy.\end{aligned}$$