Math 2374 - Quiz 1

Name: ____________________________________

Section: ______

1. (40 points) Find the area of the parallelogram formed by the vectors $u = 3\widehat{i} + 4\widehat{j}$ and $v = \widehat{i} + 2\widehat{j}$

Solution: One possible solution is to take the absolute value of the determinant of the matrix formed by the two vectors:

$$\begin{aligned}\left|\det\begin{pmatrix}3 & 1\\ 4 & 2\end{pmatrix}\right| &= |3\cdot 2 - 1\cdot 4|\\ &= 2.\end{aligned}$$

One could also take the length of the cross product of u and v seen as vectors in $\mathbb{R}^3$:

$$\begin{aligned}u\times v = \det\begin{pmatrix}\widehat{i} & \widehat{j} & \widehat{k}\\ 3 & 4 & 0\\ 1 & 2 & 0\end{pmatrix} &= \widehat{i}(4\cdot 0 - 0\cdot 2) - \widehat{j}(3\cdot 0 - 0\cdot 1) + \widehat{k}(3\cdot 2 - 4\cdot 1)\\ &= 2\widehat{k},\end{aligned}$$

and the length of $2\widehat{k}$ is just 2.

2. (60 points) Find the distance from the point $P = (1, 2, 3)$ to the plane with equation:

$$x + y + z = 0.$$

Solution: One can just apply the formula from the book:

$$\begin{aligned} \text{distance} &= \frac{|Ax_1 + Bx_2 + Cx_3 + D|}{\sqrt{A^2 + B^2 + C^2}} \\ &= \frac{1 \cdot 1 + 1 \cdot 2 + 1 \cdot 3}{\sqrt{3}} \\ &= \frac{6}{\sqrt{3}} = 2\sqrt{3}. \end{aligned}$$

If you don't remember the formula you can reason as follows: take an arbitrary point in the plane, say $Q = (0, 0, 0)$.

The distance from P to the plane will be the length of the projection of the vector QP onto the direction of the normal vector of the plane $n = (1, 1, 1)$.

So we can use the dot product to compute this:

$$\begin{aligned} \left\|\text{Proj}_n(QP)\right\| &= \left\| \frac{QP \cdot n}{\|n\|} \frac{n}{\|n\|} \right\| \\ &= \frac{|(1, 2, 3) \cdot (1, 1, 1)|}{\|(1, 1, 1)\|} \\ &= \frac{6}{\sqrt{3}} = 2\sqrt{3}. \end{aligned}$$

See also `https://mathinsight.org/distance_point_plane` for more on this.