Math 2374 - Midterm 3

Name: ______________________________

Section: ________

Student ID: ______________________________

Signature: ______________________________

- This exam consists of 6 pages (including this one) and 5 questions.
- Do not give numerical approximations to quantities such as $\sin 5$, π, or $\sqrt{2}$. However, you should simplify $\cos \pi/4 = \sqrt{2}/2$, $e^0 = 1$, and so on.
- Show your work, in a reasonably neat and coherent way, in the space provided. All answers must be justified by valid mathematical reasoning, including the evaluation of definite and indefinite integrals. To receive full credit on a problem, you must show enough work so that your solution can be followed by someone without a calculator.
- Mysterious or unsupported answers will not receive full credit. Your work should be mathematically correct and carefully and legibly written.
- A correct answer, unsupported by calculations, explanation, or algebraic work will receive no credit; an incorrect answer supported by substantially correct calculations and explanations will receive partial credit.
- Full credit will be given only for work that is presented neatly and logically; work scattered all over the page without a clear ordering will receive from little to no credit.

1	2	3	4	5	Total

1. (20 points) Compute the quadratic Taylor approximation of the function

$$f(x) = e^{x^2 - y^2}$$

at the point $(0, 0)$.

2. (20 points) Consider the function

$$f(x) = x^3 + y^3 - x^2 - y^2$$

- (10 points) Find all critical points (but do not classify them).

- (10 points) Classify the critical points $A = (0, 0)$ and $B = (2/3, 0)$.

3. (20 points) Use Stokes' theorem to compute

$$\iint_S \operatorname{curl}(-y\vec{i} + x\vec{j} + xyz\vec{k}) \cdot dS,$$

where S is the portion of the unit sphere given by $x^2 + y^2 + z^2 = 1$ and $z \geq -3/4$, oriented with the outward unit normal.

4. (20 points) Use the divergence theorem to compute

$$\iint_S \left((x+y)\vec{i} + (y-x)\vec{j} + z\vec{k}\right) \cdot dS,$$

where S is the unit sphere with the outward unit normal orientation.

5. (20 points) Compute the area of the portion of surface given by

$$x^2 - y^2 = z \quad \text{and} \quad x^2 + y^2 \leq 1.$$