Math 2374 - Final exam

Name: ______________________________

Student ID: ______________________________

Signature: ______________________________

- This exam consists of 16 pages (including this one) and 11 questions. Please count and make sure you have 11 questions.
- Do not give numerical approximations to quantities such as $\sin 5$, π, or $\sqrt{2}$. However, you should simplify $\cos \pi/4 = \sqrt{2}/2$, $e^0 = 1$, and so on.
- Show your work, in a reasonably neat and coherent way, in the space provided. All answers must be justified by valid mathematical reasoning, including the evaluation of definite and indefinite integrals. To receive full credit on a problem, you must show enough work so that your solution can be followed by someone without a calculator.
- Mysterious or unsupported answers will not receive full credit. Your work should be mathematically correct and carefully and legibly written.
- A correct answer, unsupported by calculations, explanation, or algebraic work will receive no credit; an incorrect answer supported by substantially correct calculations and explanations will receive partial credit.
- Full credit will be given only for work that is presented neatly and logically; work scattered all over the page without a clear ordering will receive from little to no credit.

Problem	Score	Max Score
1		10
2		10
3		10
4		10
5		10
6		10
7		10
8		10
9		10
10		10
11		10
Total		110

1. Let

$$f(x, y) = e^{x-y} + xy,$$

and suppose $g : \mathbb{R}^2 \to \mathbb{R}^2$ is some differentiable function such that $g(0, 0) = (1, 2)$, and

$$Dg(0, 0) = \begin{pmatrix} 1 & 2 \\ 3 & 4 \end{pmatrix}.$$

Compute the gradient of $f \circ g$ at $(0, 0)$.

2. Given the function

$$f(x, y) = x^2 - y^2 + \sin(2\pi xy),$$

compute the tangent plane at the point $(3, 2, 5)$ and write it in the form $Ax + By + Cz + D = 0$.

3. Given

$$f(x,y) = 4 - x^2 - y^2 + x^3 + y^3.$$

- Find the *unit* vector pointing in the direction in which f grows fastest at the point $(1,1)$.

- Compute the directional derivative of f at the point $(1,1)$ in the direction of the vector $(3,4)$.

4. Given

$$f(x, y) = x^2 + 5x + 5 + 2y^2 - 7y - 3xy.$$

Show that the point $(-1, 1)$ is a critical point and classify it as a local minimum, local maximum, saddle point, or something else.

5. Compute

$$\int_0^1 \int_x^1 e^{y^2}\, dy\, dx$$

by changing the order of integration to $dx\, dy$.

6. Compute (using a triple integral) the volume of the region in space bounded by the cylinder $x^2 + y^2 = 1$, the plane $z = -1$ and the plane $z = x$.

7. Let γ be any curve starting at the point $A = (1, 2)$ and ending at the point $B = (3, 2)$. Compute

$$\int_{\gamma} F \cdot ds,$$

where $F = \big(2xy + y + 2\pi y \cos(2\pi xy), x^2 + x + 1 + 2\pi x \cos(2\pi xy)\big)$.

8. Compute the scalar surface integral

$$\iint_S \frac{x^2 + y^2}{\sqrt{1 + x^2 z^2 + y^2 z^2}} \, dS,$$

where S is the surface given by

$$z = e^{xy} \quad \text{and} \quad x^2 + y^2 \leq 1.$$

9. Let S be the surface which lies in intersection of the graph of $z = xy$ with the infinite cylinder $x^2 + y^2 \leq 1$. Let C be its boundary oriented *clockwise* when seen from above. Compute

$$\int_C (x, y, z^2 + y^2 - x^2) \cdot ds$$

using Stokes' theorem. (No credit will be given for any other method. Make sure to carefully explain whether your parametrization is orientation preserving or not, and use the correct sign based on this.)

10. Let B be the box bounded by the planes $x = 0$, $y = 0$, $z = 0$, $x = 1$, $y = 2$, $z = 3$. Use the divergence theorem to compute the flux integral

$$\iint_S \left(x^2 + e^{yz}, y^2 + \sin(xz), z^2 + e^{x^2+y^2}\right) \cdot dS,$$

where S is the boundary of B with outward pointing unit normal.

11. Let u and v be any two vectors in $\mathbb{R}^3$. Explain why

$$(u - v) \cdot \big((u + 2v) \times (3u + 2v)\big) = 0.$$

Scratch paper

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