Practice problems for midterm 2

These are some problems for you to practice for mideterm 2. The midterm will probably have problems close to some of these.

1 Double and triple integrals

1. Compute the area of the region bounded by
$$x = 0, \quad y = 0, \quad y = x^2, \quad \text{and } x = y^2$$
2. Compute the are of the region above the X axis, above the line $y = x$, and inside $x^2 + y^2 = 1$.
3. Compute
$$\iint_A e^{x^2+y^2}\, dxdy$$
where A is the annulus given by $1 \le x^2 + y^2 \le 4$.
4. Compute
$$\iint_E e^{x^2+2y^2}\, dxdy$$
where E is the elliptical annulus given by $1 \le x^2 + 2y^2 \le 4$.
Hint: first make a change of variables to end up with a domain like in the previous problem.
5. Consider the cylinder with axis parallel to the Z axis, bottom at $z = 0$, top at $z = 10$, and whose base is a circle centered at $(3, 2, 0)$ of radius 3. Intersect this cylinder with a plane $z = -\frac{1}{2}(x - 3)$. Compute the volume of the wedge below this plane and inside the cylinder.
6. Compute
$$\iiint_R x^2 + y^2 + z^2\, dxdydx$$
where R is the region inside the unit sphere and below the cone $0 \le z \le \sqrt{x^2 + y^2}$.

2 Path integrals

7. Set-up but do not evaluate the length of the curve parametrized by
$$\gamma(t) = (t, t^2, t^3) \quad 0 \le t \le 1.$$

8. Let γ be the curve given by $y = \log x$ for $e \leq x \leq e^2$. Compute

$$\int_\gamma e^{2y}\, ds.$$

3 Vector fields

9. Suppose that $f(x, y) = x^2 + g(y)$. Find a function g so that

$$\operatorname{div} \nabla f = 0.$$

10. Let V be the vector field given by

$$V(x, y) = (x, y).$$

Find a flow line $c(t)$ so that $c(0) = (1, 2)$.

11. Find a $Q(x, y)$ which will make the vector field $(y \cos x, Q(x, y))$ conservative.

4 Line integrals

12. Compute the line integral of $V(x, y) = (x, -y)$ along the unit circle oriented positively.
13. Compute the line integral of $V(x, y) = (x, y)$ along the unit circle oriented positively.
14. Compute the line integral of $V(x, y) = (y, x)$ along the unit circle oriented positively.
15. Compute the line integral of $V(x, y) = (-y, x)$ along the unit circle oriented positively.
16. Let c be the piecewise curve given by the triangle ABC (oriented positively), where $A = (0, 0)$, $B = (3, 0)$, $C = (1, 4)$. Compute the line integral

$$\int_c x\, dx + y\, dz + yz\, dy$$

17. Let $V = (y \cos x, \sin x + y)$. Find a potential for V (so, a function f whose gradient is V).
18. $V = (y \cos x + x^3, \sin x - y)$. Compute the line integral

$$\int_c V \cdot s,$$

where c is the straight line going from $(0, 0)$ to $(\pi/2, 1)$.

5 Green's theorem

19. Let γ be the boundary of the ellipse $x^2 + 3y^2 = 1$ oriented positively. Use Green's theorem to evaluate

$$\int_\gamma (3y, 10x) \cdot ds$$

20. Let γ be the piecweise curve given by: a straight line starting at $(0,0)$ and ending at $(1/\sqrt{2}, 1/\sqrt{2})$, goes counterclockwise along the unit circle all the way to $(-1,0)$, and then goes back to $(0,0)$. Compute

$$\int_\gamma (y^3 + \cos(x^2)dx + (-x^3 + \sin(y^2))dy.$$

Note that there was a typo in the previous version which made it very hard to do.